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this remaining one giving a value of $S \not\equiv 0 \pmod{p}$. Thus are obtained $p^3(p-1)^3(p^2-p-1)$ new sets(V).

This exhausts the cases to consider. The sum of II, III, IV, and V is h , the order of the automorph of G . This sum is

$$h = p^3(p-1)^2(p+1)(p^2+p-1).$$

MECHANICS.

184. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

A sphere, radius a , rests between two parallel thin perfectly rough rods A and B in the same horizontal plane at a distance apart equal to $2c$; the sphere is turned about A until its center is very nearly vertically over A ; it is then allowed to fall back. Prove that it will rock between A and B if $10c^2 < 7a^2$; also, that θ_r , the angle through which it will turn after the r th impact is given by the equation $\cos\theta_r = \frac{\sqrt{(a^2 - c^2)}}{a} + \frac{a - \sqrt{(a^2 - c^2)}}{a} \left(1 - \frac{10c^2}{7a^2}\right)^{2r}$.

I. Solution by PROFESSOR WILLIAM HOOVER, Ph. D., Athens, Ohio.

Let ω' = the angular velocity of the sphere about its center at any time t' from the beginning of the first stage of motion; k, k_1 the radii of gyration about the center and a point of the surface, respectively; m = its mass, ω_1, ω_2 , etc., the angular velocities about the center just after the first, second, etc., impacts; V' = the linear velocity of the center just before the first impact, $p = (a^2 - 2c^2)/a$ = the perpendicular from a point of contact upon the direction of V' , θ' = the angle the radius makes with the vertical corresponding to ω' , and g = the acceleration of gravity. Then for the first impact, the moment of angular momentum is

$$mk_1^2 \omega_1 = (k^2 \omega' + V' p) m \text{ (1), or, } \omega_1 = \frac{k^2 \omega' + V' p}{k_1^2} \text{ (2).}$$

For the first stage of motion,

$$mk_1^2 \frac{d^2 \theta'}{dt'^2} = mg a \sin \theta' \text{ (3).}$$

Integrating (3), noticing that initially, $d\theta'/dt' = 0$, $\cos\theta' = 1$,

$$mk_1^2 \frac{d\theta'^2}{dt'^2} = 2mga(1 - \cos\theta') \text{ (4).}$$

When the first stage of motion is completed,

$$\cos\theta' = \frac{\sqrt{(a^2 - c^2)}}{a}, \text{ and (4) then is, since } k_1^2 = \frac{7}{5}a^2 \text{ (5),}$$

$$\omega'^2 = \frac{10g}{7a} \left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right) \dots\dots\dots (6).$$

$$\therefore V' = a\omega' = a\sqrt{(2ag)} \sqrt{\frac{5}{7a^2} \left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right)} \dots\dots\dots (7).$$

(2) then gives, since $k^2 = \frac{2}{5}a^2$,

$$\omega_1^2 = 2ag \left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^{\frac{2}{5}} \div \frac{7}{5}a^2 \dots\dots\dots (8).$$

For rocking, the factor $1 - (10c^2/7a^2)$ must be positive, or $10c^2 < 7a^2$.

For the second stage of motion, θ_1, t_1 corresponding to θ', t' in the first,

$$mk_1^2 \frac{d^2\theta_1}{dt_1^2} = mg \sin\theta_1 \dots\dots\dots (9).$$

Integrating (9) and noticing that when $\frac{d\theta_1}{dt_1} = \omega_1$, $\cos\theta_1 = \frac{\sqrt{(a^2 - c^2)}}{a}$,

$$mk_1^2 \frac{d\theta_1^2}{dt_1^2} = 2mg \left[\left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^2 + \sqrt{(a^2 - c^2)} - a \cos\theta_1 \right] \dots\dots\dots (10).$$

When the motion ceases for the first time, $d\theta_1/dt_1 = 0$, and then (10) gives

$$\cos\theta_1 = \frac{\sqrt{(a^2 - c^2)}}{a} + \left(\frac{a - \sqrt{(a^2 - c^2)}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^2 \dots\dots\dots (11).$$

For the backward motion, θ'', t'', ω'' corresponding to θ', t', ω' , we have

$$mk_1^2 \frac{d^2\theta''}{dt''^2} = mg \sin\theta'' \dots\dots\dots (12).$$

Integrating (12), and noticing that, when $\cos\theta'' = \cos\theta_1$, $d\theta''/dt'' = 0$, we have

$$\frac{7}{5}ma^2 \cdot \omega''^2 = 2mg \left[a \cos\theta'' - \sqrt{(a^2 - c^2)} + \left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^2 \right] \dots\dots\dots (13).$$

When

$$\cos\theta'' = \frac{\sqrt{(a^2 - c^2)}}{a}, \text{ (13) gives } \omega''^2 = 2 \left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^2 g \div \frac{7}{5}a^2 \dots\dots\dots (14).$$

Now displacing ω_1, ω', V' by ω_2, ω'', V'' , we find

$$\omega_2^2 = 2ag \left(1 - \frac{\sqrt{(a^2 - c^2)}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^4 \div \frac{7}{5}a^2 \dots\dots\dots (15);$$

and by a process entirely analogous to that commencing with equation (9), we find

$$\cos\theta_2 = \frac{\sqrt{a^2 - c^2}}{a} + \left(\frac{a - \sqrt{a^2 - c^2}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right) \dots \dots (16),$$

and generally, by inference,

$$\cos\theta_r = \frac{\sqrt{a^2 - c^2}}{a} + \left(\frac{a - \sqrt{a^2 - c^2}}{a} \right) \left(1 - \frac{10c^2}{7a^2} \right)^{2r} \dots \dots (17).$$

I. Solution by the PROPOSER.

Let C be the center of the sphere at the instant of impact, ω_{r-1} , ω_r , the angular velocities of the sphere just before and just after the r th impact, respectively; $\angle ACB = 2\beta$, $k^2 =$ radius of gyration, then $(a^2 + k^2)\omega_r = (k^2 + a^2 \cos 2\beta)\omega_{r-1}$. Now, $\sin\beta = c/a$, $k^2 = \frac{2}{5}a^2$.

$$\therefore \frac{7}{5}a^2\omega_r = a^2 \left(\frac{7}{5} - \frac{2c^2}{a^2} \right) \omega_{r-1}. \quad \therefore \frac{\omega_r}{\omega_{r-1}} = 1 - \frac{10c^2}{7a^2}.$$

$\therefore 1 - \frac{10c^2}{7a^2}$ must always be positive. Let $\omega =$ angular velocity just before first impact. By the principle of energy,

$$\begin{aligned} \frac{1}{2}(a^2 + k^2)\omega^2 &= ga(1 - \cos\beta), \\ \frac{1}{2}(a^2 + k^2)\omega_r^2 &= ga(\cos\theta_r - \cos\beta). \end{aligned}$$

$$\therefore \frac{\omega_r^2}{\omega^2} = \frac{\cos\theta_r - \cos\beta}{1 - \cos\beta}. \quad \cos\theta_r = \cos\beta + (1 - \cos\beta) \left(\frac{\omega_r}{\omega} \right)^2.$$

$$\cos\beta = \frac{\sqrt{a^2 - c^2}}{a}, \quad \frac{\omega_r}{\omega} = \left(1 - \frac{10c^2}{7a^2} \right)^r.$$

$$\therefore \cos\theta_r = \frac{\sqrt{a^2 - c^2}}{a} + \frac{a - \sqrt{a^2 - c^2}}{a} \left(1 - \frac{10c^2}{7a^2} \right)^{2r}.$$

PROBLEMS FOR SOLUTION.

ALGEBRA.

247. Proposed by PROFESSOR G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Find the sum, to n terms, of

$$1 + \frac{n}{2} + \frac{n(n+2)}{2.4} + \frac{n(n+2)(n+4)}{2.4.6} + \dots \dots$$